Circle constant is a turn

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The mathematical constant pi (π) is given as the ratio $\pi = C/D \approx 3.14$, where C is a circle's circumference and D is its diameter. I agree with Bob Palais that the definition of the well known mathematicel constant π represents a wrong definition of the circle constant. Please, see "π Is Wrong!" by Bob Palais: (`http://www.math.utah.edu/~palais/pi.pdf`)
The definition of π is not only a pedagogical problem. I think that there are not only the practical reasons for the new definition of the circle constant represented by 1 turn $= C/r$. And everybody knows what is a turn! But there is *a deeper theoretical meaning* behind the new circle constant definition C/r. *I will try to explain my viewpoint.*

The statement $\pi = C/D$ is only a fraction of the circle circumference and the circle diameter. If you call it a circle constant, it is still only a mathematical constant and nothing else. π **is only a number**! It is not even an angle value. π **is without a measurement unit!**

> The statement $\pi = C/D$ represents only a **calculation number** $\pi \approx 3.14$ for all circles, and nothing else! **This statement has not directly any other meaning!**

Why the ratio C/D of the number π is a constant and radius-invariant? **The statement $\pi = C/D$ hides any possible physical meaning!** This is a shallow understanding of the circle constant. I see that I have to start my explanations by some most basic ideas.

Suppose that you are a child, and you still do not know what is a circle. But you know very well what is a distance, without any definition. Something is in a smaller or in a bigger distance. The distance or the line length describes how far apart points (or objects) are. This is a basic concept. You know also what is a a rotation and what is a turn. An angle represents only a rotation about a point. There is the physical meaning of it. An angle means to turn in a different direction. There is also the radius-invariance of an angle. In other words, an angle is the same for every radius. Thus, an angle is represented by two rays and by a common point called vertex. We do not use any length for the definition of an angle, and we do not use any angle for the definition of a length.

> The angle and the length are independent from each other.

They are the physical dimensions. They can be used to define our mathematical coordinates. Only a mathematical abstract point has not any dimensions.

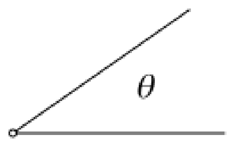

Figure 1: An angle is radius-invariant and it is given by θ (theta).

Every geometrical definition is based on the *physical dimensions* length and angle.

Let us have the Cartesian coordinates and the Pythagorean theorem:

$$x^2 + y^2 = r^2$$

You never write it as $x^2 + y^2 = (D/2)^2$! **A circle is defined by its radius.**

The radius of a circle is a circle property.

It is obvious that the radius of a circle r must be used for all calculations. The circle points can not be defined by a diameter. *A diameter is only a convenient value for a measurement.* Its value can be used to get a radius $r = D/2$.

One turn means a full rotation about a point. A turn is a constant and a natural reference angle. Thus, the angle unit degree is defined simply as a part of a turn. In this case, a turn has 360 parts or 360°.

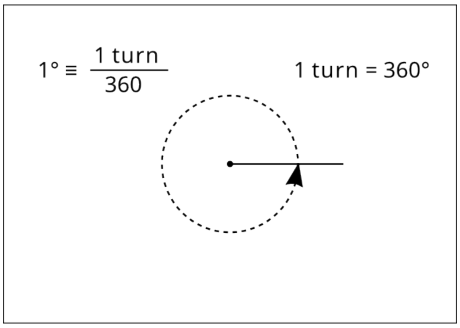

Figure 2: The unit degree represents only a part of a turn.

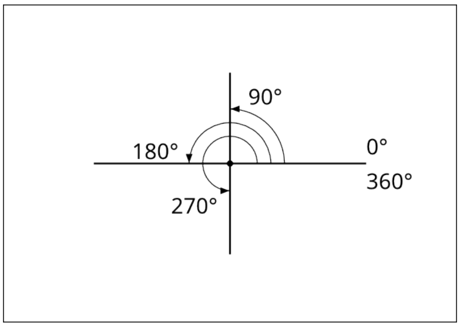

Figure 3: Some special angles.

We also already know that the arc lengths grows in proportion to the radius.

$$\frac{s_1}{r_1} = \frac{s_2}{r_2}$$

where s is the arc length and r is the radius of a circle.

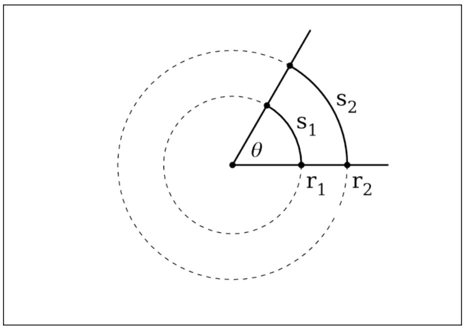

Figure 4: Radius-invariance of an angle.

If the angle is defined as:

$$\boxed{\theta \equiv \frac{s}{r}} \tag{1}$$

there is also the radius-invariance of the angle.

The ratio of the **arc length** of a circle and its **radius** represents an angle value in radian.

If $s = r$ we get the angle unit $\theta = s/r = r/r = 1\,\mathrm{rad}$. For us is also very important the turn value in radian! The turn is only a special angle and it is always constant. Let us denote it by θ_0 (theta-zero). **The circumference of a circle represents the arc length of a whole circle.** It has only a special name. If the arc length is the circumference of the circle (or $s = C$), we get

$$\theta_0 = \frac{s}{r} = \frac{C}{r} = 1\,\mathrm{turn} \tag{2}$$

Thus, the turn value C/r **is an angle value in radian**.

If a turn is given by $\theta_0 = C/r$, the fraction C/r must have also a constant value. The value C/r is a constant **angle value**. Thus, it is independent from a circle radius!

There are always about 6.28 radians in a full circle.

$$\frac{C}{r} = 6.2831853\ldots \text{ a constant value in radians} \tag{3}$$

See the definition:

$$\boxed{\theta_0 \equiv \frac{C}{r}} \tag{4}$$

The constant θ_0 represents a turn in radian.
θ_0 is already by definition an angle value.

An angle θ can be given in an angle unit. A turn is an angle given by θ_0. $\theta_0 = 1$ (1 implicit turn) is the angle of a full rotation. And the symbol θ_0 can represent generally a turn in every angle unit. A turn has always a constant value in degree $\boxed{\theta_0 = 360°}$ or a constant value in the natural unit radian $\boxed{\theta_0 \approx 6.28\,\text{rad}}$.

$$\theta_0 = \frac{C}{r} = 6.2831853\ldots \text{ (the turn value in radian)} \tag{5}$$

There is the meaning of the constant θ_0 (θ_0 is an angle value, and it represents a turn), there is also the value of it C/r, and it is constant (one turn is always one turn, regardless of a circle radius). This interpretation is like the definition of the speed of light. The speed of light is also a speed. Its value is the speed of light in vacuum and it is constant regardless of inertial reference frame.
The constant C/r represents a turn. This constant has not any mystery.

The immutability of the value C/r means
the obvious fact:
A turn is a constant angle,
regardless of a circle radius.

Thus, we get always a constant turn value in radian or in degree: $\theta_0 = 360°$. We can write also:

$$\frac{\theta_0}{2} = 180°, \quad \frac{\theta_0}{4} = 90°, \quad \ldots$$

This notation is very easy to understand! A turn is an angle, and a part of a turn is also an angle. The fraction values $\theta_0/2$, $\theta_0/4$, ... are the angle values in radian. An angle value can be compared with an angle value. **The angle unit degree is defined as a part of a turn.** You can see the definition of a degree even from the unit conversion given by $1° = \theta_0/360$. It is also $90° = 90 \cdot 1° = 90 \cdot \theta_0/360 = \theta_0/4$. You see that $\theta_0/4 = 90°$. It is much better if a quarter of a turn is given by $\theta_0/4$. Thus, $\theta_0/4 = 360°/4 = 90°$.

The statement $\pi = C/D$
does not defines an angle!
π is defined by a diameter, and **it has not an angle unit**. It is only a calculation number ≈ 3.14 and nothing else!

Number 1 is a number, and 1 rad is an angle value. The constant π is a number 3.14159..., and π rad (or 3.14 rad) is an angle value (the unit radian must be explicit). The number π has not a measurement unit. The notation $\pi/2 = 90°$ is wrong! Also the value 2π is only a number. And π rad refers to a semicircle! You can not define a semicircle without a turn and a circle! The circle is a natural form and a turn has a simple physical meaning. A turn represents a full rotation about a point. *The most important thing is that the turn value $\theta_0 \equiv C/r$ is well-defined, and it explains also the physical nature of the circle constant.* **The circle constant θ_0 is an angle value and it has the measurement unit radian**. You can use the angle value in radian $\theta_0/2$. The fundamental difference is that π is **a number**, and $\theta_0/2$ is **a constant value in radian**. The definition $\pi \equiv C/D$ is the problem!

The angle value θ_0 has a measurement unit. But the number π has not any unit! $\pi \equiv C/D$ is not an angle value! π is not one-half θ_0!

If a circle is a 2-dimensional shape made by drawing a curve that is always (*in any direction*) the same distance r from a center, how we can draw it without a turn (*without a full rotation about a point*)?! From the eq. 4 we get:

$$\boxed{C = \theta_0 \cdot r} \tag{6}$$

It's that simple. This is like drawing a circle by a compass. $C = \theta_0 \cdot r$ represents directly the definition of the circle.

A circle is a line of **a full rotation** in **a distance** from the center of the rotation. *A circle is defined by a turn and a radius!* A circle can be defined only by its properties. A turn and a radius are the circle properties!

This is what we assume when we make our calculations. But, if we assume it, **why we have not an explicit notation for it?** Compare it with $\pi = C/D$ and $C = \pi \cdot D$! Even if you write $C = 2\pi \cdot r$ (?!), π is a nonsense. A turn is something fundamental. A circle has the rotational symmetry and it can be defined explicitly only **by a turn and a radius**. A turn and a radius are the essential concepts of the circle definition. And the unit radian of a turn in θ_0 is also not a problem. It is an SI derived unit and can be expressed as m/m. Thus, we get the circumference unit $[C] = [\theta_0][r] = \text{rad} \cdot \text{m} = (\text{m}/\text{m}) \cdot \text{m} = \text{m}$.

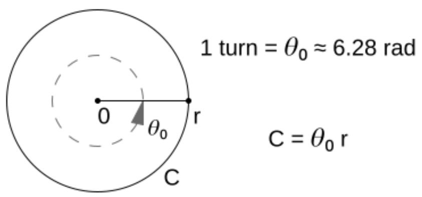

Figure 5: Drawing a circle by a turn. A turn is given by $\theta_0 = C/r$.

If A is the area of a circle, do you see a *turn* in $A = \pi r^2$?! I can see here the radius of a circle, but I do not see the second physical dimension of the circle. The number π does not represents a dimension. A turn represents a rotation and the second dimension of the circle! It is given by the well-defined **angle value** θ_0. See the equations below:

$$C = \theta_0 \cdot r \quad \text{and} \quad A = \frac{1}{2}\,\theta_0 \cdot r^2 \tag{7}$$

You can see directly that the angle (it is given by a turn) and the length (it is given by a radius) are the physical dimensions of a circle. This is really a geometrical view of a circle. **A circle is a geometrical form, but it has a physical meaning!** It is a natural form. The circle constant is used also for the physical and the technical descriptions. Thus, the circle constant must have also a physical meaning. The circumference of a circle is the length of the circle line. $\boxed{C = \theta_0 \cdot r}$ A length is a physical value. θ_0 has also a physical meaning. An angle value has a measurement unit and it is a physical value!

A turn $\theta_0 = C/r$ is a **physical constant**.

The circle constant definition can not be an arbitrary definition! **Mathematics may not use the abstractions to hide any physical meaning!** Even if you write $C = 2\pi \cdot r$, you assume that **a turn** is given by **the number** 2π! But the turn value $\boxed{\theta_0 = C/r \approx 6.28\,\text{rad}}$ **is** a natural or a physical constant! A craftsman can still use a diameter. Don't worry! The statement $r = D/2$ remains unaffected. Regardless of wether you accept the turn value $\theta_0 = C/r$ as the circle constant or not, the turn is in the definition of the circle.

The definition of the circle presuppose the turn. If the turn is a constant, it is obviously the circle constant.

On the other hand, if the circle constant is independent from a circle radius and it is also a circle property, it is a turn. **A turn is independent from a circle radius**, and **it is a circle property**. See the relation:

$$\theta_0 = \frac{C}{r} = \text{const} \quad \leftrightarrow \quad C = \theta_0 \cdot r \tag{8}$$

It is so simple relation! A turn is defined by a circle, and a circle is defined by a turn. But they are not equal. We get the unit circle by $r = 1$. If we do not care about the radius of a circle (if $r = 1$), the circle is only a graphical representation of a turn. A turn is a full rotation *in any distance* about a point! *It is independent from a circle radius*! And a circle is a line of **a turn** in a given distance r (it has a radius). $\boxed{C = \theta_0 \cdot r}$ The turn θ_0 **is** the circle constant!